Lecture 4 - Detailed Solutions to Examples

Example 3

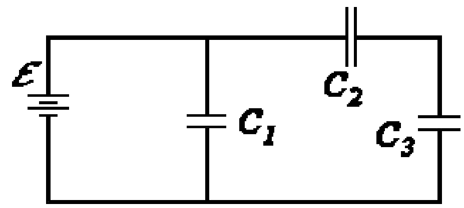

Figure 1

The best way to see which of the given answers are in fact correct is to solve for the equivalent capacitance and then work the problem backwards to solve for the voltages and charges at each of the intermediate points.

First combine the two capacitors, C_2 and C_3, that are in series with each other.

$$\frac{1}{C_4} = \frac{1}{C_2} + \frac{1}{C_3}$$
$$C_4 = \frac{C_2\,C_3}{C_2 + C_3}$$

The circuit now looks like

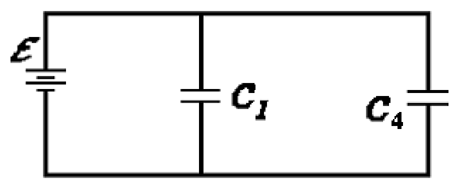

Figure 2

We now combine the two capacitors that are in parallel with each other, C_1 and C_4.

$$C_5 = C_1 + C_4 = C_1 + \frac{C_2\,C_3}{C_2 + C_3}$$

We see from this that the equivalent capacitor, C_5, is in fact larger than C_1.

Therefore the suggested answer f) is correct.

The circuit now looks like

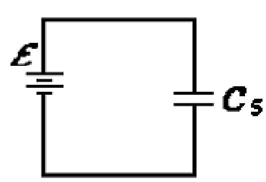

Figure 3

The voltage across C_5 is the same as the battery, E. The voltage across C_5 is also the same as that across C_1 and C_4 in Figure 2. So we then have that

$$E = V_1 \qquad and \qquad E = V_4$$

Therefore the suggested answer d) is correct.

The voltage drops across C_2 and C_3 in Figure 1 have to add to the voltage drop across C_4 in Figure 2, so answer c) is incorrect. And since V_2 and V_3 add to V_4 and since V_4 is the same as V_1, V_1 is greater than V_2, answer e) is also incorrect.

We now have only to deal with the charges and their arrangements on the various capacitors. The charge that will be on the equivalent charge, Q_5, will be split between C_1 and C_4. The charges that will reside on capacitors C_2 and C_3 will be the same as that on C_4. From this we see that

Answer b) is correct.

We then also have that answer a) is incorrect.

Example 5

The capacitance of a parallel set of conductors of area A and separation d is given by

$$C = \varepsilon_0 \frac{A}{d}$$

Attaching the capacitor to a battery of voltage V gives the capacitor a charge Q which is given by

$$Q = C\,V = \varepsilon_0 \frac{A\,V}{d}$$

Now the battery is disconnected from the capacitor, though there will be still be a voltage difference, V, across the plates of the capacitor. The energy stored in the capacitor is given by

$$U = \frac{1}{2}\frac{Q^2}{C}$$

The electric field between the plates is given by

$$E = V\,d$$

You now pull the plates of the capacitor further apart to a separation of d_1.
Several of the quantities will change, some will not.

Since the capacitor is not connected to a source, **the charge on the capacitor will not change** as there is no method by which charge can either enter or leave the capacitor.

The capacitance will change as the separation between the plates changes. The new capacitance is given by

$$C_1 = \varepsilon_0 \frac{A}{d_1}$$

Using that $C\,d = \varepsilon_0 A$ we then have

$$C_1 = \frac{d}{d_1} C$$

Since $d_1 > d$ we have that $C_1 < C$, **the capacitance decreases.**

Since the capacitance changes, but the charge does not, the voltage changes. The voltage in terms of the capacitance and the charge is given by

$$V_1 = \frac{Q}{C_1} = \frac{Q}{\frac{d}{d_1} C} = \frac{d_1}{d}\frac{Q}{C} = \frac{d_1}{d} V$$

The voltage across the capacitance increases.

What happens to the potential energy stored in the capacitor? Initially we had

$$U = \frac{1}{2} Q\,V$$

After the plates are pulled apart, we now have

$$U_1 = \frac{1}{2} Q\,V_1 = \frac{1}{2} Q \frac{d_1}{d} V = \frac{d_1}{d} U$$

The final potential energy is larger than the initial potential energy. The normal tendency for the plates would be to move towards each other. You had to do work to pull the plates apart and this work went into the potential energy.

What happens to the electric field? The initial electric field between the two plates is given by

$$E = \frac{V}{d}$$

After the plates are pulled apart the electric field is given by

$$E_1 = \frac{V_1}{d_1} = \frac{1}{d_1}\frac{d_1}{d} V = \frac{V}{d} = E$$

The electric field does not change. The electric field only depends upon the charge density on the plates.

Example 6

As in Example 5 we start from the following:

The capacitance of a parallel set of conductors of area A and separation d is given by

$$C = \varepsilon_0 \frac{A}{d}$$

Attaching the capacitor to a battery of voltage V gives the capacitor a charge Q which is given by

$$Q = C\,V = \varepsilon_0 \frac{A\,V}{d}$$

Now the battery is disconnected from the capacitor, though there will be still be a voltage difference, V, across the plates of the capacitor. The energy stored in the capacitor is given by

$$U = \frac{1}{2}\frac{Q^2}{C}$$

The electric field between the plates is given by

$$E = V\,d$$

We now pull the plates apart but we leave the battery attached this time. Since the battery is still attached **the voltage across the capacitor does not change**.

To determine what happens to the capacitance we go back to the formula for parallel plate capacitors

$$C_1 = \varepsilon_0 \frac{A}{d_1}$$

Using that $C\,d = \varepsilon_0 A$ we then have

$$C_1 = \frac{d}{d_1} C$$

Since $d_1 > d$ we have that $C_1 < C$, **the capacitance decreases.**

The charge in the capacitor is now given by

$$\begin{aligned} Q_1 &= C_1 V \\ &= \frac{d}{d_1} C\,V \\ &= \frac{d}{d_1} Q \end{aligned}$$

The charge is now less. **The charge on the capacitor decreases.**

What happens to the potential energy stored in the capacitor? Initially we had

$$U = \frac{1}{2} Q\,V$$

After the plates are pulled apart, we now have

$$U_1 = \frac{1}{2} Q_1\,V = \frac{1}{2}\frac{d}{d_1} Q\,V = \frac{d}{d_1} U$$

The potential energy decreases.

What happens to the electric field between the plates? The electric field is given by

$$E_1 = \frac{V}{d_1} = \frac{E\,d}{d_1} = \frac{d}{d_1} E$$

The electric field also decreases.

Example 7

Before the dielectric is inserted into capacitor C_2 we have the following after the battery of voltage V has been removed:

$$\begin{aligned} C_1 &= C_2 = C \\ V_1 &= V_2 = V \\ Q_1 &= Q_2 = Q = C\,V \end{aligned}$$

By inserting the dielectric of dielectric constant K, the value of the capacitance increases for the second capacitor. The second capacitance is then

$$C_2^{'} = K\,C$$

The voltage across the second capacitor is then given by

$$V_2^{'} = \frac{Q}{C_2^{'}} = \frac{Q}{K\,C} = \frac{1}{K} V_1$$

Since K is larger than 1, the new voltage across the second capacitor is less than originally, or the voltage across the second capacitor is less than the voltage across the first capacitor.