Physics 232 Final Exam
May 4, 2004

1. **An object of height 2cm is located 50cm to the left of a diverging lens of focal length 50cm. To the right of this lens, at a distance of 5 cm, is a converging lens of focal length 15cm. Describe completely the final image.**

STEP 1

We first transport the 2cm object through the diverging lens

$$\begin{aligned} \frac{1}{S}+\frac{1}{S'} &= \frac{1}{f} \\ \frac{1}{50}+\frac{1}{S'} &= \frac{1}{-50} \\ \frac{1}{S'} &= -\frac{1}{50}-\frac{1}{50}=-\frac{2}{50}=-\frac{1}{25} \\ S' &= -25cm \end{aligned}$$

This is a **VIRTUAL IMAGE** that is to the **LEFT** of the diverging lens.
The magnification of this *intermediate* image is given by

$$m_1 = -\frac{S'}{S} = -\frac{-25}{50} = \frac{1}{2}$$

This image is **half the height** but with the **same orientation**.

STEP 2

We now transport this image through the second lens. The first immge is at a distance of 25 + 5 = 30cm from the second lens. This is treated as a **real object, positive object distance,** since to get to the second lens we go left to right which is the same direction as the ongoing light.

$$\begin{aligned} \frac{1}{S}+\frac{1}{S'} &= \frac{1}{f} \\ \frac{1}{30}+\frac{1}{S'} &= \frac{1}{15} \\ \frac{1}{S'} &= \frac{1}{15}-\frac{1}{30}=\frac{2}{30}-\frac{1}{30}=\frac{1}{30} \\ S' &= 30cm \end{aligned}$$

This is a **REAL IMAGE** that is to the **RIGHT** of this converging lens. This is also the final image, which therefore is **REAL**.
The magnification for this step is given by

$$m_2 = -\frac{S'}{S} = -\frac{30}{30} = -1$$

This image is the **same height** but **inverted** with respect to the first image.
The total magnification is just the product of the two individual magnifications.

$$\begin{aligned} m_{total} &= m_1 m_2 \\ &= \left(\frac{1}{2}\right)(-1) \\ &= -\frac{1}{2} \end{aligned}$$

So the final image is -$\frac{1}{2} * 2 = -1cm$ high. It is **inverted** with respect to the orginal orientation of the original object.

2. **A particle of mass 0.1kg is executing simple harmonic motion. It is found to be oscillating with a frequency of 15.92 Hz. It is also found to be at x = 7.5cm with a velcoity of v = 1299 cm/s at a certain time.**

   (a) **What is the spring constant?**

   (b) **What is the total energy of the system?**

   (c) **What is the maximum amplitude.**

   (d) **What is the equation of motion that describes the system? Be sure to evaluate all appropriate constants.**

a) The angular frequency is related to the spring constant and mass by

$$\omega = \sqrt{\frac{k}{m}}$$

with the angular frequency related to the frequency by

$$\omega = 2\,\pi\,f = 100\,rads/\sec$$

We therefore have

$$\begin{aligned} 2\,\pi\,f &= \sqrt{\frac{k}{m}} \\ k &= 4\,\pi^2 f^2\,m \\ &= 4\,\pi^2 (15.92)^2 (0.1) \\ &= 1000.567\;Nts/m \end{aligned}$$

b) The total energy is given by the sum of the kinetic energy and the potential energy.

$$\begin{aligned} E_{total} &= KE + PE \\ &= \frac{1}{2} m\,v^2 + \frac{1}{2} k\,x^2 \\ &= 0.5(0.1)(1299/100)^2 + 0.5(1000.567)(0.075)^2 \\ &= 8.437 + 2.814 \\ &= 11.25\,joules \end{aligned}$$

c) The maximum amplitude can be obtained by setting the total energy equal to the potential energy

$$\begin{aligned} E_{total} &= PE = \frac{1}{2} k\,A^2 \\ A &= \sqrt{\frac{2E_{total}}{k}} \\ &= \sqrt{\frac{2(11.25)}{1000.567}} \\ &= 0.15\,m = 15\,cm \end{aligned}$$

d) The equation of motion is given by

$$x = A\cos(\omega\, t + \delta)$$

The only constant we don't have is the phase angle. This can be obtained from the boundary conditions that are given, namely that $x = 7.5cm$ and $v = 1299\, cm/\sec$ which we assume to be at time t = 0. The velocity equation is given by

$$v = \frac{dx}{dt} = -\omega A\sin(\omega\, t + \delta)$$

The equation we have at t = 0 are then

$$\begin{aligned} 0.075 &= 0.15\cos\delta \\ 12.99 &= -(100)(0.15)\sin\delta \end{aligned}$$

We simplify these a bit

$$\begin{aligned} \sin\delta &= -0.866 \\ \cos\delta &= 0.5 \end{aligned}$$

or

$$\begin{aligned} \tan\delta &= -1.832 \\ \delta &= -60.0^0 = -\frac{\pi}{3} \end{aligned}$$

The equation of motion is then

$$x = 0.15\cos(100t - \frac{\pi}{3})$$

3. **You are driving your car at 50 ft/sec. You notice that a car is approaching you from the front at 60 ft/sec. The other car sounds its horn, which operates at a frequency of 200 Hz.**

   (a) **What is the frequency that you hear?**

   (b) **The car then passes you. What is the frequency that you now hear?**

a) The situation looks like

The positive sense for velocities, listener to source, is left to right. So your velocity is positive, while the velocity of the other car is negative. The necessary equation is

$$\begin{aligned}
\frac{f_{listener}}{V_{sound} + V_{listener}} &= \frac{f_{source}}{V_{sound} + V_{source}} \\
\frac{f_{listener}}{1100 + 50} &= \frac{200}{1100 - 60} \\
f_{listener} &= \frac{1150}{1040} 200 \\
&= 221.15 \, Hz
\end{aligned}$$

b) Now the situation looks like

Now the positive sense for velocities is from right to left. So your vleocity is now negative, while the other car's velocity is now positve.

$$\begin{aligned}
\frac{f_{listener}}{V_{sound} + V_{listener}} &= \frac{f_{source}}{V_{sound} + V_{source}} \\
\frac{f_{listener}}{1100 - 50} &= \frac{200}{1100 + 60} \\
f_{listener} &= \frac{1050}{1160} 200 \\
&= 181.03 \, Hz
\end{aligned}$$

4. **Two audio loudspeakers are being driven, in phase, at a frequency of 680 Hz. They are located on the y-axis with one being at $y = 0.75m$ while the other is at $y = -0.75m$. You are standing on the x-axis far from the speakers.**

   (a) **At what angle above the horizontal do you encounter the 1^{st} minimum?**

   (b) **One of the channels of the amplifier starts to drift and develops a phase differnece of 45^0 with respect to the other channel. Now at what angle above the horizontal do hear the 1^{st} maximum?**

Using the velocity of sound to be 340 m/s we find the wavelength for these audio waves to be

$$\begin{aligned} v &= \lambda f \\ \lambda &= \frac{v}{f} = \frac{340}{680} = 0.5m \end{aligned}$$

a) The phase difference between the two speakers consists of two parts - one due to any path difference for the two sound waves and the second being any inherent phase difference between the speakers

$$\delta = \delta_{pathdifference} + \delta_{inherent}$$

Since the speakers are in phase we only need to consider the phase difference due to the path difference

$$\begin{aligned} \delta &= 2\pi\left(\frac{PathDifference}{\lambda}\right) \\ &= 2\pi\left(\frac{d\sin\theta}{\lambda}\right) \end{aligned}$$

where d is the distance between the speakers and θ is the angle above the horizontal. Since we are looking for a minimum this phase difference needs to be an odd integer multiple of π.

$$\begin{aligned} \delta &= 2\pi\left(\frac{d\sin\theta}{\lambda}\right) = (2n+1)\pi \\ \frac{d\sin\theta}{\lambda} &= \frac{2n+1}{2} \\ \sin\theta &= \frac{\lambda}{d}\frac{2n+1}{2} \end{aligned}$$

The first minimum will be for n = 1. We then have

$$\begin{aligned} \sin\theta &= \frac{\lambda}{d}\frac{2n+1}{2} \\ &= \frac{0.5}{1.5}\frac{1}{2} = \frac{1}{6} \\ \theta &= 9.59^0 \end{aligned}$$

b) We now have an inherent phase difference between the speakers of 45^0. We therefore have to include the second term for the total phase difference.

$$\begin{aligned}\delta &= 2\pi\left(\frac{PathDifference}{\lambda}\right) + \left(\frac{PhaseAngle}{180}\right)\pi \\ &= 2\pi\left(\frac{d\sin\theta}{\lambda}\right) + \left(\frac{45}{180}\right)\pi\end{aligned}$$

Since we are looking for a maximum, the total phase difference needs to be an integer multiple of 2π.

$$2\pi\left(\frac{d\sin\theta}{\lambda}\right) + \left(\frac{45}{180}\right)\pi = n(2\pi)$$

We set n = 1 for the first maximum and divide through by 2π and obtain

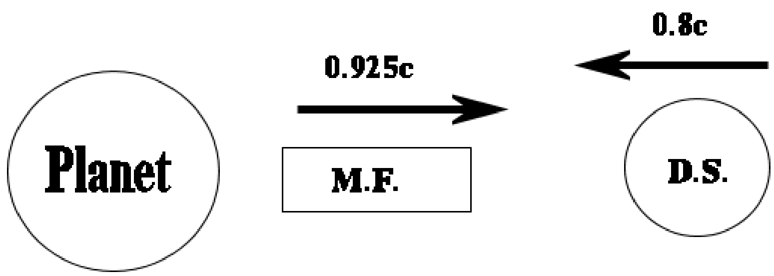
$$\begin{aligned}\frac{d\sin\theta}{\lambda} + \frac{1}{8} &= 1 \\ \frac{d\sin\theta}{\lambda} &= \frac{7}{8} \\ \sin\theta &= \frac{7}{8}\frac{\lambda}{d} = \frac{7}{8}\frac{0.5}{1.5} = 0.292 \\ \theta &= 16.98^0\end{aligned}$$

5. A rebel observer on the ice planet Hoth notices that the Empire's Death Star is approaching the planet at a speed of 0.8c. The observer also sees that the Milenium Falcon is racing recklessly towards the Death Star at a speed of 0.925c as measured by the observer on the planet.

   (a) According to Darth Vader who is on the Death Star, how fast is the Millenium Falcon approaching the Death Star?

   (b) Han Solo, who is on the Millenium Falcom, notices that it takes him 120 seconds to overtake the Death Star. According to him, how far apart where the two spaceships initially?

   (c) According to the observer on the ice planet, how far apart were the two spaceships initially?



a) Since we want the velocity of the Millenium Falcon as seen by the Death Star we attach the unprimed coordinate system to the Ice Planet and the primed system to the Death Star. The relative velocity of the two coordinate systems is -0.8c. $v_x = 0.925c$.

$$\begin{aligned} v'_x &= \frac{v_x - v}{1 - (v\,/\,c^2)\; v_x} \\ &= \frac{0.925c - (-0.8c)}{1 - (-0.8c\,/\,c^2)\,0.925c} \\ &= \frac{1.725c}{1 + 0.74} = \frac{1.725c}{1.74} \\ &= 0.9914c \end{aligned}$$

b) Han Solo sees the Death Star approaching him with the same velocity as the Death Star sees him approaching. So the distance traveled by the Death Star is given by

$$\begin{aligned} L_{H.S.} &= v'_x\, t \\ &= (0.9914)(3 \times 10^8)(120) \\ &= 3.569 \times 10^{10} m \end{aligned}$$

c) Since Han Solo measures the time interval at the same point in his coordinate system, he has the **proper time**. Every one on the ice planet sees a longer time interval.

$$T = \frac{T_{proper}}{\sqrt{1-(v/c)^2}}$$

For this part of the problem v is the velocity of the Millenium Falcon with respect to the Ice Planet, $v = 0.925c$.

$$\begin{aligned} T &= \frac{120}{\sqrt{1-0.925^2}} = (2.632)(120) \\ &= 315.82\,\text{sec} \end{aligned}$$

According to the oberver on the Ice Planet the relative velocities of the Millenium Falcon and the Death Star is given by

$$v_{rel} = 0.925c - (-0.8c) = 1.725c$$

The initial separation as seen by the observer is then

$$\begin{aligned} Separation &= v_{rel}T \\ &= (1.725)(3\times10^8)(315.82) \\ &= 1.634\times10^{11} m \end{aligned}$$

6. **For the reaction**

$${}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

The following mass values may be of use: $M({}^{4}_{2}He) = 4.002603\,u$; $M({}^{14}_{7}N) = 14.003074\,u$; $M({}^{17}_{8}O) = 16.999133\,u$; $M({}^{1}_{1}H) = 1.007825\,u$; $u = 931.5\,MeV/c^2$.

(a) **Calculate the reaction energy.**

(b) **Is the reaction endothermic or exothermic?**

a) The reaction energy is given by

$$Q = (M_A + M_B - M_C - M_D)\,c^2$$

where A and B are in the incoming particles, while C and D are the outgoing particles.

We then have

$$\begin{aligned} Q &= [(4.002603 + 14.003074) - (16.999133 + 1.007825)] \times 931.5 \\ &= [18.005677 - 18.006958] \times 931.5 \\ &= -0.001281 \times 931.5 \\ &= -1.19\,MeV \end{aligned}$$

b) Since the reaction energy is negative, the reaction is **endothermic** and the incoming alpha particle needs at least this amount of kinetic energy before the reaction will take place.

7. **While on vacation you find yourself on the midway of a seaside resort wherein Planck's constant has the value of 1 joule*sec. A sideshow worker offers you $1000 if you would hold a quarter in your outstretched hand so that a sharpshooter can demostrate his ability. The quarter is 12 mm in radius. The bullet travels with a velocity of 305 m/s and has a mass of 20 grams. The radius of the bore of the rifle is 5 mm. Assume that the bullet is negligible in size. You are to stand 30 m away from the sharpshooter. Should you take the offer? A yes or no answer is insufficient.**

The position of the bulletas it exits the gun barrel is uncertain in the y-direction. This introduces an uncertainty in the momentum in the y-direction.

$$\Delta p_y \Delta y = \hbar$$

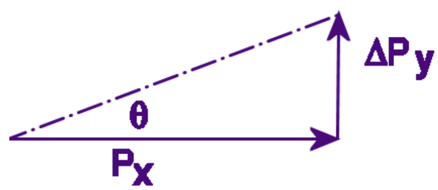
with $\Delta y = 0.005\,m$. Then

$$\begin{aligned}\Delta p_y &= \frac{1}{2\,\pi}\frac{1}{0.005} \\ &= 31.83\,\frac{kg}{m \cdot s}\end{aligned}$$

The momentum in the x-direction is given by

$$\begin{aligned}p_x &= m\,v_x \\ &= (20 \times 10^{-3})(305) \\ &= 6.1\frac{kg}{m \cdot s}\end{aligned}$$

This defines a cone of uncertainty in the direction of the bullet.



$$\begin{aligned}\tan\theta &= \frac{\Delta p_y}{p_x} \\ &= \frac{31.83}{6.1} = 5.218 \\ \theta &= \tan^{-1}(5.218) \\ &= 79.15^0\end{aligned}$$

The radius of the circle that this uncertainty describes at your position is given by

$$\begin{aligned}\frac{\Delta r}{x} &= \frac{\Delta p_y}{p_x} \\ \Delta r &= x \cdot \frac{\Delta p_y}{p_x} \\ &= (30)(5.218) \\ &= 156.54\,m\end{aligned}$$

This is much larger than the radius of the quarter, so the odds are he will miss the quarter, as the radius of the quarter is 12 mm.