

Physics 232 Exam I

Feb. 14, 2007

ID # _____ Name _____

1. A 3 kg mass is attached to a horizontal spring and is executing simple harmonic motion with an angular velocity of 14.14 radians/sec. At a given instant in time it is found to be 12 cm to the left of the equilibrium position and moving to the right with a velocity of 1.732 m/sec.

- (a) **What is the spring constant?**

The angular velocity is given by

$$\begin{aligned}\omega &= \sqrt{\frac{k}{m}} \\ k &= \omega^2 m = (14.14)^2 (3) \\ &= 600 \text{ Nts/m}\end{aligned}$$

- (b) **What is the total energy of the system?**

The total energy of the system is given by

$$\begin{aligned}E_{Total} &= \frac{1}{2} k x^2 + \frac{1}{2} m v^2 \\ &= (0.5) (600) (0.12)^2 + (0.5) (3) (1.732)^2 \\ &= 4.32 + 4.50 \\ &= 8.82 \text{ joules}\end{aligned}$$

- (c) **What is the maximum amplitude of oscillation?**

When the mass is at its maximum amplitude of motion, all of the energy is in potential energy.

$$\begin{aligned}E_{Total} &= \frac{1}{2} k A^2 \\ A &= \sqrt{\frac{2E_{Total}}{k}} = \sqrt{\frac{2(8.82)}{600}} = 0.171 \text{ meters} = 17.1 \text{ cm}\end{aligned}$$

- (d) **What is the phase angle?**

We use the conditions stated at the beginning of the problem. Since we are not told otherwise, we assume that these occur at $t = 0$. We start from

$$x = A \cos(\omega t + \delta) \text{ and } v = -\omega A \sin(\omega t + \delta)$$

At $t = 0$ we have

$$\begin{aligned}-0.12 &= 0.171 \cos(\delta) \\ \cos(\delta) &= -0.702\end{aligned}\tag{1}$$

and

$$\begin{aligned}1.732 &= -(14.14) (0.171) \sin(\delta) \\ \sin(\delta) &= -0.716\end{aligned}\tag{2}$$

We now divide equation (2) by equation (1) to get

$$\begin{aligned}\tan(\delta) &= \frac{-0.716}{-0.702} = 1.0199 \\ \delta &= 45.566\end{aligned}\tag{3}$$

The question is does this phase angle match the boundary conditions. Since the velocity is to the right and velocity equation has an inherent negative sign in it, we have to have the sine function be negative to give us a positive velocity. The phase angle we have in equation (3) does not satisfy this condition. The correct phase angle is in fact in the third quadrant, both the sine and cosine terms are negative, and is given by

$$\begin{aligned}\delta &= 180 + 45.566 \\ &= 225.565\end{aligned}$$

2 A guitar string has a length of 64 cm. and a mass of 6.16 grams. You tune the guitar so that this string is under a tension of 68 Nts.

- (a) **What is the velocity of waves on this string.**

The wave velocity is given by

$$v = \sqrt{\frac{T}{\mu}}$$

The mass per unit length is

$$\mu = \frac{m}{L} = \frac{6.16 \times 10^{-3}}{0.64} = 9.625 \times 10^{-3} \text{ kgs/m}$$

Then we have

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{68}{9.625 \times 10^{-3}}} = \sqrt{7064.94} = 84.05 \text{ m/s}$$

- (b) **What is the wavelength of the fundamental for this string?**

The allowed wavelengths are given by

$$\lambda = \frac{2L}{n}$$

The fundamental is given when $n = 1$. We then have

$$\lambda = \frac{2(64)}{1} = 128 \text{ cm} = 1.28 \text{ m}$$

- (c) **What is the frequency of the next harmonic?**

The next harmonic is $n = 2$. The wavelength is then

$$\lambda = \frac{2(64)}{2} = 64 \text{ cm} = 0.64 \text{ m}$$

The frequency is given by

$$\begin{aligned} v &= \lambda f \\ f &= \frac{v}{\lambda} = \frac{84.05}{0.64} = 131.33 \text{ Hz} \end{aligned}$$

3. **You and a friend both are holding identical tuning forks which are each vibrating at 600 Hz.**

- (a) **Your friend is moving towards you at a speed of 30 ft/sec while you are running towards your friend at a speed of 10 ft/sec. What is the frequency that you hear from your friend's tuning fork?**

The positive sense for velocities is given by the direction from the listener to the source. For this problem then positive sense is from you to your friend. Therefore your velocity is positive, while your friend's is negative. The frequency that you hear is then given by

$$\begin{aligned} f_L &= \frac{v + v_L}{v + v_S} f_S \\ &= \frac{1100 + 10}{1100 - 30} 600 = \frac{1110}{1070} 600 = (1.0371) (600) \\ &= 622.43 \text{ Hz} \end{aligned}$$

- (b) **What is the beat frequency that you hear?**

The beat frequency is given by

$$B.F. = |f_L - f_S| = 22.43 \text{ Hz}$$

4. A ray of light starts out in a slab of glass with the glass having an index of refraction 1.6. The ray of light makes an angle of 45° with respect to the normal to the surface.

- (a) What is the speed of light in the glass?

The speed of light is reduced in a medium by a factor of the index of refraction.

$$v = \frac{c}{n} = \frac{3 \times 10^8}{1.6} = 1.875 \times 10^8 \text{ m/s}$$

- (b) What is the angle the ray of light makes with respect to the normal upon exiting the glass?

We use Snell's law

$$\begin{aligned} n_a \sin \theta_a &= n_b \sin \theta_b \\ \sin \theta_b &= \frac{n_a}{n_b} \sin \theta_a \\ &= \frac{1.6}{1} \sin 45 \\ &= (1.6)(0.7071) \\ &= 1.13 \end{aligned}$$

This is an unphysical value for the sine function. Therefore the light ray is totally internally reflected!

Physics 232 Formula Sheet

Simple Harmonic Motion

$$\vec{F} = -k \vec{x}$$

$$x = A \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f$$

$$T = \frac{1}{f}$$

$$PE = \frac{1}{2} k x^2$$

$$E_{Total} = PE + KE$$

$$E_{Total} = \frac{1}{2} k A^2$$

Doppler Effect

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Periodic Motion

$$y(x, t) = A \cos(kx \pm \omega t)$$

$$y(x, t) = 2A \cos(kx) \sin(\omega t)$$

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2}$$

$$v = \lambda f$$

$$y_{net} = \sum_i y_i$$

$$P_{ave} = \frac{1}{2} \mu v \omega^2 A^2$$

Waves on a String

$$v = \sqrt{\frac{T}{\mu}}$$

$$\lambda = \frac{2L}{n} \quad n = \text{integer}$$

Light

Reflection

$$\theta_{\text{incident}} = \theta_{\text{reflected}}$$

Refraction

$$n_a \sin \theta_a = n_b \sin \theta_b$$

$$\sin \theta_c = \frac{n_a}{n_b} \quad n_a < n_b$$

Polarization

$$I = I_0 \cos^2 \phi$$

Electromagnetic Waves

$$E_{\text{max}} = c B_{\text{max}}$$

$$I = S_{\text{ave}} = \frac{1}{2} \epsilon_0 c E^2$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Physical Constants

$$c = 3 \times 10^8 \text{ m/s}$$

$$v_{\text{sound}} = 1100 \text{ ft/sec}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.109 \times 10^{-31} \text{ kg}$$

$$m_p = 1.672 \times 10^{-27} \text{ kg}$$

Useful Geometry

Circle

$$\text{Area} = \pi r^2$$

$$\text{Circumference} = 2\pi r$$

Sphere

$$\text{Surface Area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

Cylinder

$$\text{Lateral Area} = 2\pi r L$$

$$\text{Volume} = \pi r^2 L$$